

1. National Research Coordinator's Comments (English)

- [00:00:09](#) This lesson covers a part of the compulsory eighth-grade math program with advanced demands. It took place in the canton of Ticino, in the Italian part of Switzerland.
- [00:00:14](#) The pupils already know the Pythagorean theorem and can apply it to two-dimensional polygons. The goal of the lesson is to push the pupils to use it with solids.
- [00:00:57](#) The methodology used by the teacher is based on situated teaching (1990, Guy Brousseau, France), with important moments of development (1999, Bruno D'Amore, Italy) in which pupils take responsibility for their own learning and are free to act in groupwork. This methodology is explicitly taught to math teachers and supported by the math experts in the canton of Ticino.
- The situation presented by the teacher gives no hint about the use of the Pythagorean Theorem: pupils are asked to place the longest possible straw into the Swiss postal boxes (which have a rectangular shape).
- [00:09:42](#) The first difficulty pupils have to overcome is to understand that the best solution is to place the straw in the diagonal of the solid. Some pupils take the diagonal of the base (bottom) of the box as the solution. The teacher encourages them to find a better solution and helps them in their thinking process.
- [00:11:54](#) The second step is organizing the calculation. The Pythagorean theorem needs to be applied twice, in the correct sequence. The teacher never suggested to use the Pythagorean theorem; the pupils themselves had this idea. Some of them may have guessed its use, others followed into the steps of their peers.
- [00:13:13](#) The calculation is eased by the worksheet prepared by the teacher (which may be considered even too detailed). The teacher makes sure the pupils fill out the worksheet correctly.
- [00:36:19](#) As everyone has finished their calculations, the teacher interrupts the groupwork to share the results using the transparencies the pupils have prepared.
- [00:40:26](#) Not only the correctness of the results is controlled at this point, the teacher also enhances the discussion about the easiest and quickest solution method, stressing the mathematical way of writing. Using the solution of one pupil, she shows how to elegantly write the whole calculation with just one mathematical expression. Reflecting on the solution process is a didactical habit teachers in the canton of Ticino are encouraged to apply. The goal is to teach the pupils to make metacognitive reflections about their learning (1996, Xavier Roegiers, Belgium).
- [00:43:27](#) In the last part of the lesson the teacher gives those pupils who haven't finished yet time

to fill out the worksheet. Meanwhile, she assigns several exercises to other students, in which the Pythagorean theorem is applied to different concrete situations.

In the short time of one lesson it is not possible for all pupils to acquire a good level of competence in applying the Pythagorean theorem to solids. As the official math program suggests, this activity will be re-proposed at regular intervals of time (based on the principle of "spiral" teaching).

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Finally, it is important to see how this lesson opens further possibilities:

First, the change from the numerical solution to the algebraic form. The way the pupils have written their calculations is already highly formal - a numerical expression with use of parentheses and symbols for the power and the root - and therefore a preparation to the algebraic version. In the canton of Ticino, this change into the algebraic form is prepared beginning in the sixth grade and gradually developed until the ninth grade.

Second, a discussion could be started about mathematical methods that would allow to decide immediately which of the initially given boxes is the best choice (without having to calculate the length of the diagonal). This could be done, for example, using the sum of the length of the sides, or one could suggest to use their product. A computer-based research could be organized, tables could be made, and one could spend some time thinking and exploring...